INTERMEDIATE MACROECONOMICS

MALTHUSIAN MODEL OF GROWTH

22. OVERVIEW OF THE MALTHUSIAN MODEL

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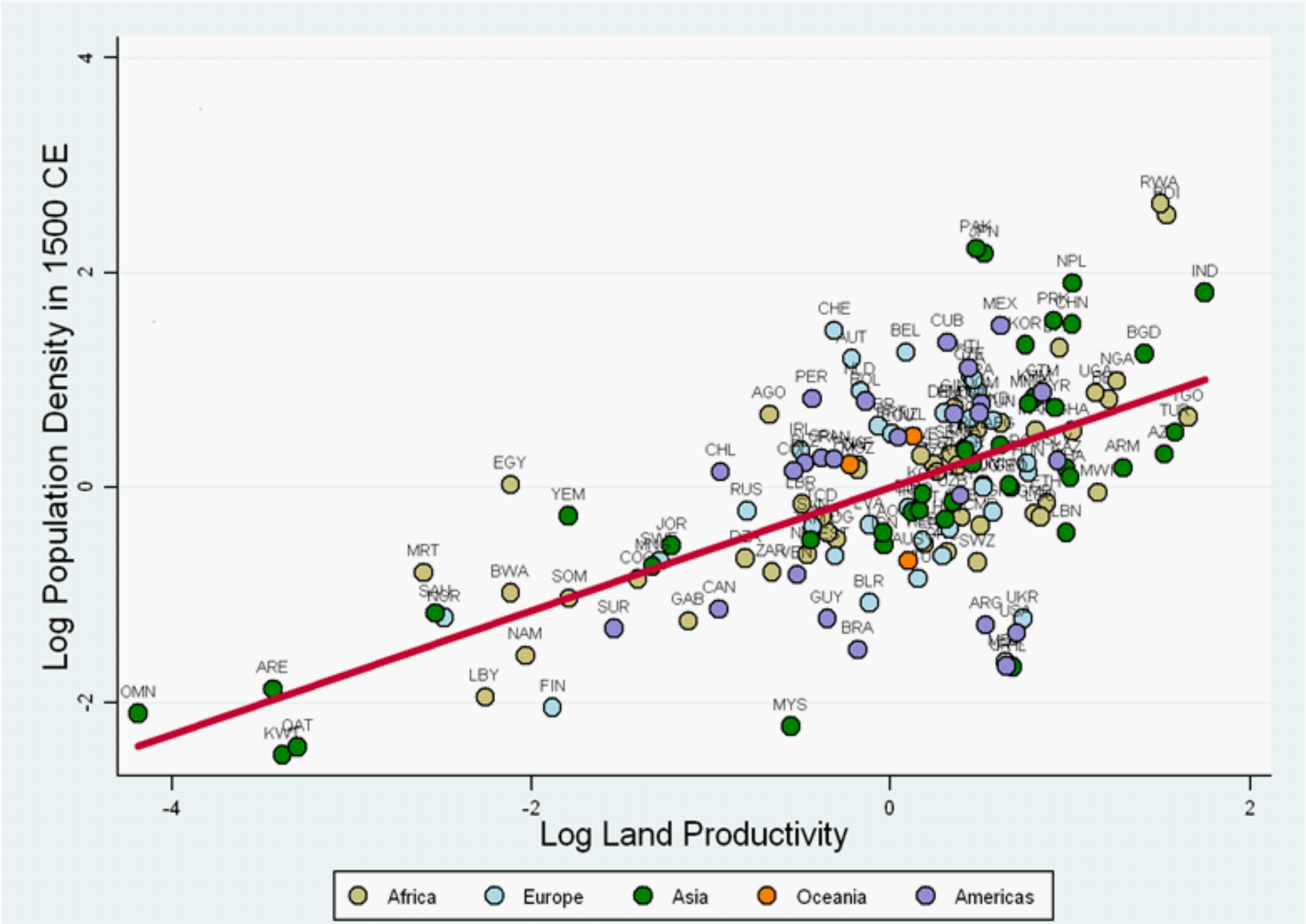
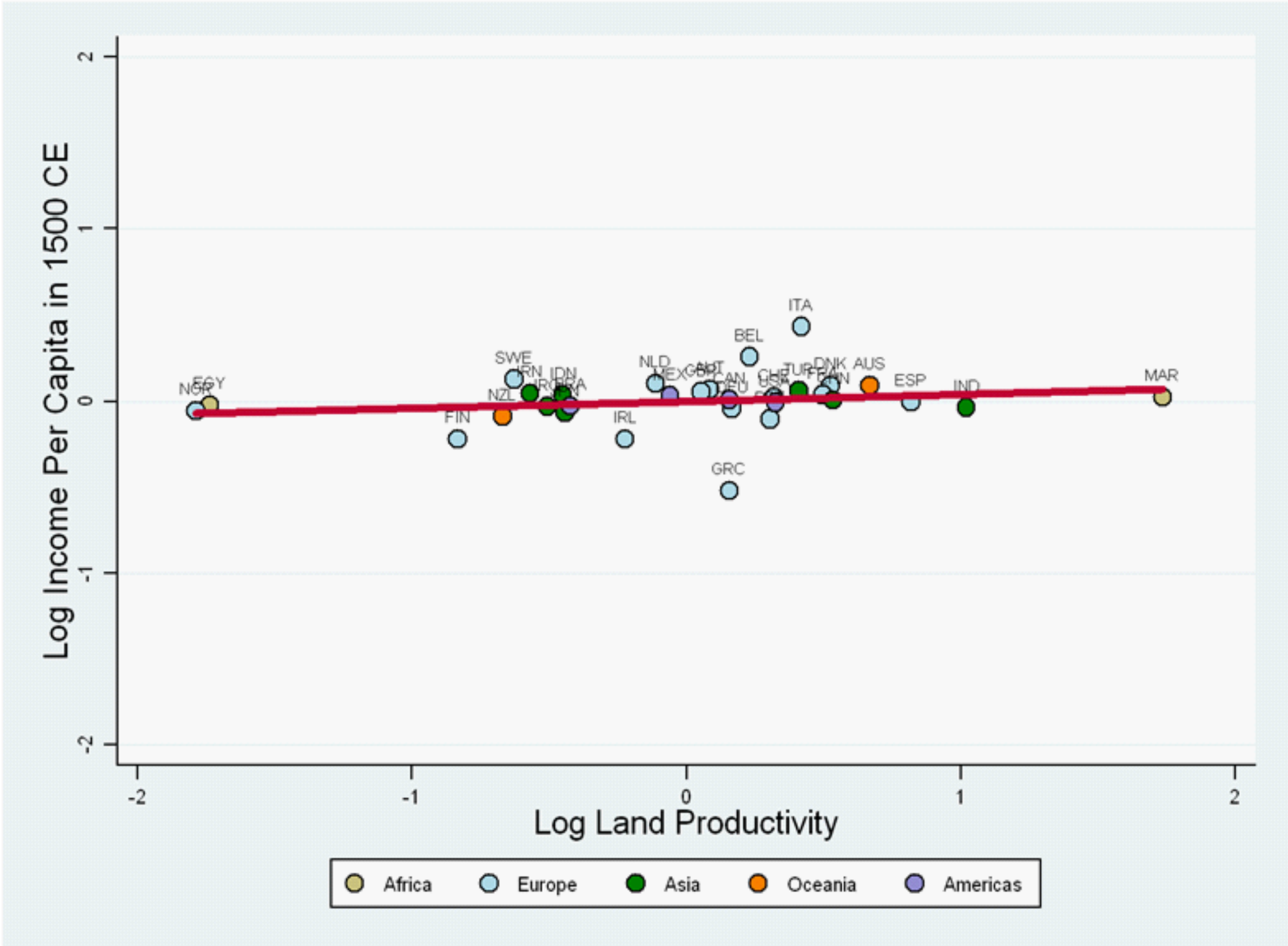
US GROWTH AFTER INDUSTRIAL REVOLUTION

- between 1870 and today, US growth = 2% per year
  - especially fast since 1950
- US real GDP: GDP(2014) = 46 × GDP(1890)
  - much of it driven by population growth
- US real GDP/person: GDP(2014) = 9 × GDP(1890)
  - due to better technology: inventions, production and management techniques, infrastructure, legal and political institutions

HIGH LAND PRODUCTIVITY LEADS TO HIGH POPULATION…



…BUT HAS NO EFFECT ON INCOME PER CAPITA!



THE MALTHUSIAN MODEL

- objective: understand how technology advancements may lead to higher population but same living standards

- overlapping generations (two generations live simultaneously)
  - people are born, live two periods, and die
  - at the beginning of each period, new people are born
  - at the end of each period, existing people die
- people produce and consume one good: food
- two factors of production: labor + land

FACTORS OF PRODUCTION

- land X: fixed over time
- technology A: fixed over time
  - percentage of arable land, soil quality, climate, cultivation and irrigation methods, crops available
- labor L: evolves over time
  - determined by the rate of population growth
  - in turn determined by workers' fertility

COBB-DOUGLAS PRODUCTION FUNCTION

- $Y(t)=(A\ \ X)^{\alpha} \times L(t)^{1-\alpha}$
  - A: technology
  - X: amount of land
  - L(t): number of workers in period t
  - Y(t): output in period t
  - $0 < \alpha < 1$: parameter of the production function
- more workers, more land, better technology yield more output

OUTPUT PER WORKER

- y(t): output produced by a worker in period t
  - $y(t) = Y(t) / L(t) = [A X / L(t)]^{\alpha}$
- more land and better technology lead to a higher output per worker
- but more workers lead to lower output per worker
  - this is because the amount of land is fixed, so with more workers, each worker farms a smaller plot and therefore produces less output

THE LIFE OF WORKERS

- workers live 2 periods in households organized around one parent
- childhood period: the worker is a child and does not work
  - a child consumes an amount p of food that she receives from her one parent
- adulthood period: the worker is a parent and works
  - a parent decides how many children to have
  - a parent then produces food, consumes some of it, and give some of it to her children

COBB-DOUGLAS UTILITY FUNCTION

- $u(t) = n(t)^{\beta} \times c(t)^{1-\beta}$
  - u(t): utility enjoyed by a worker
  - n(t): number of children in the worker's household
  - c(t): worker's consumption
  - $0 < \beta < 1$: parameter of the utility function describing how much workers value having children
- a worker enjoys consuming more food and having more children

WORKER'S BUDGET CONSTRAINT

- the budget constraint says that the worker splits her food production between herself and her children

- $y(t) = p \times n(t) + c(t)$
  - y(t): food produced by a worker
  - c(t): food consumed by a worker
  - p × n(t): food consumed by the worker's n(t) children

- for a worker, having more children brings utility but reduces personal consumption

NUMBER OF CHILDREN THAT WORKERS CAN AFFORD

- a worker must split her food production between herself and her children: $y(t) = p \times n(t) + c(t)$

- hence, for a production $y(t)$ and a consumption $c(t)$, a worker can only afford:
  - $n(t) = [y(t) - c(t)] \ / \ p$ children

- a worker can support fewer children when
  - the worker or the children consume more
  - output per worker is lower